

EXERCISE - 01
CHECK YOUR GRASP
SELECT THE CORRECT ALTERNATIVE (ONLY ONE CORRECT ANSWER)

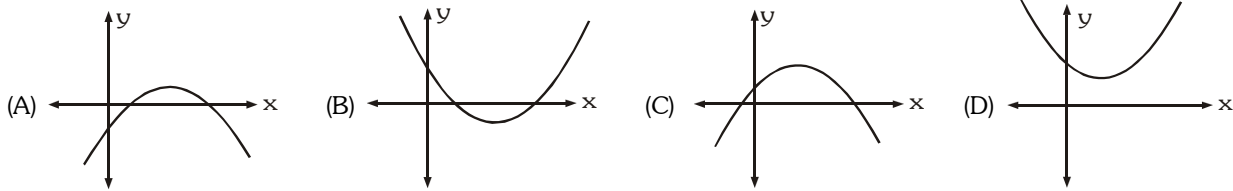
- The roots of the quadratic equation $(a + b - 2c)x^2 - (2a - b - c)x + (a - 2b + c) = 0$ are -
 (A) $a + b + c$ & $a - b + c$ (B) $1/2$ & $a - 2b + c$
 (C) $a - 2b + c$ & $1/(a + b - 2c)$ (D) none of these
- If the A.M. of the roots of a quadratic equation is $\frac{8}{5}$ and A.M. of their reciprocals is $\frac{8}{7}$, then the quadratic equation is -
 (A) $5x^2 - 8x + 7 = 0$ (B) $5x^2 - 16x + 7 = 0$ (C) $7x^2 - 16x + 5 = 0$ (D) $7x^2 + 16x + 5 = 0$
- If $\sin \alpha$ & $\cos \alpha$ are the roots of the equation $ax^2 + bx + c = 0$ then -
 (A) $a^2 - b^2 + 2ac = 0$ (B) $a^2 + b^2 + 2ac = 0$
 (C) $a^2 - b^2 - 2ac = 0$ (D) $a^2 + b^2 - 2ac = 0$
- If one root of the quadratic equation $px^2 + qx + r = 0$ ($p \neq 0$) is a surd $\frac{\sqrt{a}}{\sqrt{a} + \sqrt{a-b}}$, where $p, q, r; a, b$ are all rationals then the other root is -
 (A) $\frac{\sqrt{b}}{\sqrt{a} - \sqrt{a-b}}$ (B) $a + \frac{\sqrt{a(a-b)}}{b}$
 (C) $\frac{a + \sqrt{a(a-b)}}{b}$ (D) $\frac{\sqrt{a} - \sqrt{a-b}}{\sqrt{b}}$
- A quadratic equation with rational coefficients one of whose roots is $\tan\left(\frac{\pi}{12}\right)$ is -
 (A) $x^2 - 2x + 1 = 0$ (B) $x^2 - 2x + 4 = 0$ (C) $x^2 - 4x + 1 = 0$ (D) $x^2 - 4x - 1 = 0$
- $ax^2 + bx + c = 0$ has real and distinct roots α and β ($\beta > \alpha$). Further $a > 0$, $b < 0$ and $c < 0$, then -
 (A) $0 < \beta < |\alpha|$ (B) $0 < |\alpha| < \beta$ (C) $\alpha + \beta < 0$ (D) $|\alpha| + |\beta| = \left|\frac{b}{a}\right|$
- If the roots of $(a^2 + b^2)x^2 - 2b(a + c)x + (b^2 + c^2) = 0$ are equal then a, b, c are in
 (A) A.P. (B) G.P. (C) H.P. (D) none of these
- If $a(b - c)x^2 + b(c - a)x + c(a - b) = 0$ has equal root, then a, b, c are in
 (A) A.P. (B) G.P. (C) H.P. (D) none of these
- Let $p, q \in \{1, 2, 3, 4\}$. Then number of equation of the form $px^2 + qx + 1 = 0$, having real roots, is
 (A) 15 (B) 9 (C) 7 (D) 8
- If the roots of the quadratic equation $ax^2 + bx + c = 0$ are imaginary then for all values of a, b, c and $x \in \mathbb{R}$, the expression $a^2x^2 + abx + ac$ is -
 (A) positive (B) non-negative
 (C) negative (D) may be positive, zero or negative
- If x, y are rational number such that $x + y + (x - 2y)\sqrt{2} = 2x - y + (x - y - 1)\sqrt{6}$, then
 (A) x and y cannot be determined (B) $x = 2, y = 1$
 (C) $x = 5, y = 1$ (D) none of these

12. Graph of the function $f(x) = Ax^2 - BX + C$, where

$$A = (\sec\theta - \cos\theta)(\operatorname{cosec}\theta - \sin\theta)(\tan\theta + \cot\theta),$$

$$B = (\sin\theta + \operatorname{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2 - (\tan^2\theta + \cot^2\theta) \text{ \& }$$

$C = 12$, is represented by



13. The equation whose roots are the squares of the roots of the equation $ax^2 + bx + c = 0$ is -

(A) $a^2x^2 + b^2x + c^2 = 0$ (B) $a^2x^2 - (b^2 - 4ac)x + c^2 = 0$

(C) $a^2x^2 - (b^2 - 2ac)x + c^2 = 0$ (D) $a^2x^2 + (b^2 - ac)x + c^2 = 0$

14. If $\alpha \neq \beta$, $\alpha^2 = 5\alpha - 3$, $\beta^2 = 5\beta - 3$, then the equation whose roots are α/β & β/α , is

(A) $x^2 + 5x - 3 = 0$ (B) $3x^2 + 12x + 3 = 0$ (C) $3x^2 - 19x + 3 = 0$ (D) none of these

15. If α, β are the roots of the equation $x^2 - 3x + 1 = 0$, then the equation with roots $\frac{1}{\alpha - 2}, \frac{1}{\beta - 2}$ will be

(A) $x^2 - x - 1 = 0$ (B) $x^2 + x - 1 = 0$ (C) $x^2 + x + 2 = 0$ (D) none of these

16. If $x^2 - 11x + a$ and $x^2 - 14x + 2a$ have a common factor then 'a' is equal to

(A) 24 (B) 1 (C) 2 (D) 12

17. The smallest integer x for which the inequality $\frac{x-5}{x^2+5x-14} > 0$ is satisfied is given by -

(A) -7 (B) -5 (C) -4 (D) -6

18. The number of positive integral solutions of the inequation $\frac{x^2(3x-4)^3(x-2)^4}{(x-5)^5(2x-7)^6} \leq 0$ is -

(A) 2 (B) 0 (C) 3 (D) 4

19. The value of 'a' for which the sum of the squares of the roots of $2x^2 - 2(a-2)x - a - 1 = 0$ is least is -

(A) 1 (B) $3/2$ (C) 2 (D) -1

20. If the roots of the quadratic equation $x^2 + 6x + b = 0$ are real and distinct and they differ by atmost 4 then the least value of b is -

(A) 5 (B) 6 (C) 7 (D) 8

21. The expression $\frac{x^2+2x+1}{x^2+2x+7}$ lies in the interval ; ($x \in \mathbb{R}$) -

(A) $[0, -1]$ (B) $(-\infty, 0] \cup [1, \infty)$ (C) $[0, 1)$ (D) none of these

22. If the roots of the equation $x^2 - 2ax + a^2 + a - 3 = 0$ are real & less than 3 then -

(A) $a < 2$ (B) $2 \leq a \leq 3$ (C) $3 < a \leq 4$ (D) $a > 4$

23. The number of integral values of m , for which the roots of $x^2 - 2mx + m^2 - 1 = 0$ will lie between -2 and 4 is -

(A) 2 (B) 0 (C) 3 (D) 1

24. If the roots of the equation, $x^3 + Px^2 + Qx - 19 = 0$ are each one more than the roots of the equation, $x^3 - Ax^2 + Bx - C = 0$, where A, B, C, P & Q are constants then the value of $A + B + C =$

(A) 18 (B) 19 (C) 20 (D) none

25. If $\alpha, \beta, \gamma, \delta$ are roots of $x^4 - 100x^3 + 2x^2 + 4x + 10 = 0$, then $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} + \frac{1}{\delta}$ is equal to -

(A) $\frac{2}{5}$ (B) $\frac{1}{10}$ (C) 4 (D) $-\frac{2}{5}$

26. Number of real solutions of the equation $x^4 + 8x^2 + 16 = 4x^2 - 12x + 9$ is equal to -
 (A) 1 (B) 2 (C) 3 (D) 4
27. The complete solution set of the inequation $\sqrt{x+18} < 2-x$ is -
 (A) $[-18, -2]$ (B) $(-\infty, -2) \cup (7, \infty)$ (C) $(-18, 2) \cup (7, \infty)$ (D) $[-18, -2]$
28. If $\log_{1/3} \frac{3x-1}{x+2}$ is less than unity then x must lie in the interval -
 (A) $(-\infty, -2) \cup (5/8, \infty)$ (B) $(-2, 5/8)$
 (C) $(-\infty, -2) \cup (1/3, 5/8)$ (D) $(-2, 1/3)$
29. Exhaustive set of value of x satisfying $\log_{|x|}(x^2 + x + 1) \geq 0$ is -
 (A) $(-1, 0)$ (B) $(-\infty, 1) \cup (1, \infty)$
 (C) $(-\infty, \infty) - \{-1, 0, 1\}$ (D) $(-\infty, -1) \cup (-1, 0) \cup (1, \infty)$
30. Solution set of the inequality, $2 - \log_2(x^2 + 3x) \geq 0$ is -
 (A) $[-4, 1]$ (B) $[-4, -3) \cup (0, 1]$ (C) $(-\infty, -3) \cup (1, \infty)$ (D) $(-\infty, -4) \cup [1, \infty)$

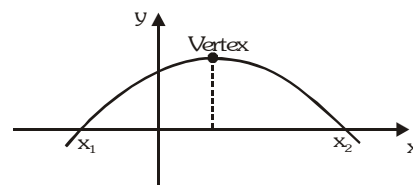
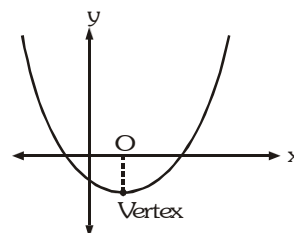
SELECT THE CORRECT ALTERNATIVES (ONE OR MORE THAN ONE CORRECT ANSWERS)

31. If α is a root of the equation $2x(2x + 1) = 1$, then the other root is -
 (A) $3\alpha^3 - 4\alpha$ (B) $-2\alpha(\alpha + 1)$ (C) $4\alpha^3 - 3\alpha$ (D) none of these
32. If $b^2 \geq 4ac$ for the equation $ax^4 + bx^2 + c = 0$, then all roots of the equation will be real if -
 (A) $b > 0, a < 0, c > 0$ (B) $b < 0, a > 0, c > 0$
 (C) $b > 0, a > 0, c > 0$ (D) $b > 0, a < 0, c < 0$
33. Let α, β be the roots of $x^2 - ax + b = 0$, where a & $b \in \mathbb{R}$. If $\alpha + 3\beta = 0$, then -
 (A) $3a^2 + 4b = 0$ (B) $3b^2 + 4a = 0$ (C) $b < 0$ (D) $a < 0$
34. For $x \in [1, 5]$, $y = x^2 - 5x + 3$ has -
 (A) least value = -1.5 (B) greatest value = 3
 (C) least value = -3.25 (D) greatest value = $\frac{5 + \sqrt{13}}{2}$
35. Integral real values of x satisfying $\log_{1/2}(x^2 - 6x + 12) \geq -2$ is -
 (A) 2 (B) 3 (C) 4 (D) 5
36. If $\frac{1}{2} \leq \log_{0.1} x \leq 2$, then -
 (A) the maximum value of x is $\frac{1}{\sqrt{10}}$ (B) x lies between $\frac{1}{100}$ and $\frac{1}{\sqrt{10}}$
 (C) x does not lie between $\frac{1}{100}$ and $\frac{1}{\sqrt{10}}$ (D) the minimum value of x is $\frac{1}{100}$

CHECK YOUR GRASP					ANSWER KEY			EXERCISE-1		
Que.	1	2	3	4	5	6	7	8	9	10
Ans.	D	B	A	C	C	B	B	C	C	A
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	B	B	C	C	A	A	D	C	B	A
Que.	21	22	23	24	25	26	27	28	29	30
Ans.	C	A	C	A	D	A	D	A	D	B
Que.	31	32	33	34	35	36				
Ans.	B,C	B,D	A,C	B,C	A,B,C	A,B,D				

EXERCISE - 02**BRAIN TEASERS****SELECT THE CORRECT ALTERNATIVES (ONE OR MORE THAN ONE CORRECT ANSWERS)**

- The equation whose roots are $\sec^2 \alpha$ & $\operatorname{cosec}^2 \alpha$ can be -
 (A) $2x^2 - x - 1 = 0$ (B) $x^2 - 3x + 3 = 0$ (C) $x^2 - 9x + 9 = 0$ (D) $x^2 + 3x + 3 = 0$
- If $\cos \alpha$ is a root of the equation $25x^2 + 5x - 12 = 0$, $-1 < x < 0$, then the value of $\sin 2\alpha$ is -
 (A) $12/25$ (B) $-12/25$ (C) $-24/25$ (D) $24/25$
- If the roots of the equation $\frac{1}{x+p} + \frac{1}{x+q} = \frac{1}{r}$ are equal in magnitude and opposite in sign, then -
 (A) $p + q = r$ (B) $p + q = 2r$
 (C) product of roots $= -\frac{1}{2}(p^2 + q^2)$ (D) sum of roots $= 1$
- Graph of $y = ax^2 + bx + c = 0$ is given adjacently. What conclusions can be drawn from this graph -
 (A) $a > 0$ (B) $b < 0$
 (C) $c < 0$ (D) $b^2 - 4ac > 0$
- If a, b, c are real distinct numbers satisfying the condition $a + b + c = 0$ then the roots of the quadratic equation $3ax^2 + 5bx + 7c = 0$ are -
 (A) positive (B) negative (C) real and distinct (D) imaginary
- The adjoining figure shows the graph of $y = ax^2 + bx + c$. Then -
 (A) $a > 0$ (B) $b > 0$
 (C) $c > 0$ (D) $b^2 < 4ac$
- If $x^2 + Px + 1$ is a factor of the expression $ax^3 + bx + c$ then -
 (A) $a^2 + c^2 = -ab$ (B) $a^2 - c^2 = -ab$ (C) $a^2 - c^2 = ab$ (D) none of these
- The set of values of 'a' for which the inequality $(x - 3a)(x - a - 3) < 0$ is satisfied for all x in the interval $1 \leq x \leq 3$
 (A) $(1/3, 3)$ (B) $(0, 1/3)$ (C) $(-2, 0)$ (D) $(-2, 3)$
- Let $p(x)$ be the cubic polynomial $7x^3 - 4x^2 + K$. Suppose the three roots of $p(x)$ form an arithmetic progression. Then the value of K , is -
 (A) $\frac{4}{21}$ (B) $\frac{16}{147}$ (C) $\frac{16}{441}$ (D) $\frac{128}{1323}$
- If the quadratic equation $ax^2 + bx + 6 = 0$ does not have two distinct real roots, then the least value of $2a + b$ is -
 (A) 2 (B) -3 (C) -6 (D) 1
- If p & q are distinct reals, then $2\{(x-p)(x-q) + (p-x)(p-q) + (q-x)(q-p)\} = (p-q)^2 + (x-p)^2 + (x-q)^2$ is satisfied by -
 (A) no value of x (B) exactly one value of x (C) exactly two values of x (D) infinite values of x
- The value of 'a' for which the expression $y = x^2 + 2a\sqrt{a^2 - 3}x + 4$ is perfect square, is -
 (A) 4 (B) $\pm\sqrt{3}$
 (C) ± 2 (D) $a \in (-\infty, -\sqrt{3}] \cup [\sqrt{3}, \infty)$



13. Set of values of 'K' for which roots of the quadratic $x^2 - (2K - 1)x + K(K - 1) = 0$ are -
 (A) both less than 2 is $K \in (2, \infty)$ (B) of opposite sign is $K \in (-\infty, 0) \cup (1, \infty)$
 (C) of same sign is $K \in (-\infty, 0) \cup (1, \infty)$ (D) both greater than 2 is $K \in (2, \infty)$
14. The correct statement is / are -
 (A) If x_1 & x_2 are roots of the equation $2x^2 - 6x - b = 0$ ($b > 0$), then $\frac{x_1}{x_2} + \frac{x_2}{x_1} < -2$
 (B) Equation $ax^2 + bx + c = 0$ has real roots if $a < 0$, $c > 0$ and $b \in \mathbb{R}$
 (C) If $P(x) = ax^2 + bx + c$ and $Q(x) = -ax^2 + bx + c$, where $ac \neq 0$ and $a, b, c \in \mathbb{R}$, then $P(x).Q(x)$ has at least two real roots.
 (D) If both the roots of the equation $(3a + 1)x^2 - (2a + 3b)x + 3 = 0$ are infinite then $a = 0$ & $b \in \mathbb{R}$
15. If $\alpha_1 < \alpha_2 < \alpha_3 < \alpha_4 < \alpha_5 < \alpha_6$, then the equation $(x - \alpha_1)(x - \alpha_3)(x - \alpha_5) + 3(x - \alpha_2)(x - \alpha_4)(x - \alpha_6) = 0$ has -
 (A) three real roots (B) no real root in $(-\infty, \alpha_1)$
 (C) one real root in (α_1, α_2) (D) no real root in (α_5, α_6)
16. Equation $2x^2 - 2(2a + 1)x + a(a + 1) = 0$ has one root less than 'a' and other root greater than 'a', if
 (A) $0 < a < 1$ (B) $-1 < a < 0$ (C) $a > 0$ (D) $a < -1$
17. The value(s) of 'b' for which the equation, $2\log_{1/25}(bx + 28) = -\log_5(12 - 4x - x^2)$ has coincident roots, is/are -
 (A) $b = -12$ (B) $b = 4$ (C) $b = 4$ or $b = -12$ (D) $b = -4$ or $b = 12$
18. For every $x \in \mathbb{R}$, the polynomial $x^8 - x^5 + x^2 - x + 1$ is -
 (A) positive (B) never positive
 (C) positive as well as negative (D) negative
19. If α, β are the roots of the quadratic equation $(p^2 + p + 1)x^2 + (p - 1)x + p^2 = 0$ such that unity lies between the roots then the set of values of p is -
 (A) ϕ (B) $p \in (-\infty, -1) \cup (0, \infty)$ (C) $p \in (-1, 0)$ (D) $(-1, 1)$
20. Three roots of the equation, $x^4 - px^3 + qx^2 - rx + s = 0$ are $\tan A, \tan B$ & $\tan C$ where A, B, C are the angles of a triangle. The fourth root of the biquadratic is -
 (A) $\frac{p-r}{1-q+s}$ (B) $\frac{p-r}{1+q-s}$ (C) $\frac{p+r}{1-q+s}$ (D) $\frac{p+r}{1+q-s}$
21. If $\log\left(\frac{x^2 - 12x + 30}{10}\right) \left(\log_2 \frac{2x}{5}\right) > 0$ then x belongs to interval -
 (A) $(\frac{5}{2}, 6 + \sqrt{6})$ (B) $(\frac{5}{2}, 6 - \sqrt{6})$ (C) $(6, 6 + \sqrt{6})$ (D) $(10, \infty)$

BRAIN TEASERS				ANSWER KEY				EXERCISE-2			
Que.	1	2	3	4	5	6	7	8	9	10	
Ans.	C	C,D	B,C	A,B,C,D	C	B,C	C	B	D	B	
Que.	11	12	13	14	15	16	17	18	19	20	
Ans.	D	C	C	A,B,C	A,B,C	A,C,D	B	A	C	A	
Que.	21										
Ans.	B,D										

EXERCISE - 03**MISCELLANEOUS TYPE QUESTIONS****TRUE / FALSE**

1. If $a, b, c \in \mathbb{Q}$, then roots of $ax^2 + 2(a+b)x - (3a+2b) = 0$ are rational.
2. The necessary and sufficient condition for which a fixed number 'd' lies between the roots of quadratic equation $f(x) = ax^2 + bx + c = 0$; ($a, b, c \in \mathbb{R}$), is $f(d) < 0$.
3. If $0 < p < \pi$ then the quadratic equation, $(\cos p - 1)x^2 + x \cos p + \sin p = 0$ has real roots.
4. The necessary and sufficient condition for the quadratic function $f(x) = ax^2 + bx + c$, to take both positive and negative values is, $b^2 > 4ac$, where $a, b, c \in \mathbb{R}$ & $a \neq 0$.

FILL IN THE BLANKS

1. If $a + b + c = 0$ & $a^2 + b^2 + c^2 = 1$ then the value of $a^4 + b^4 + c^4$ is
2. If $x^2 - 4x + 5 - \sin y = 0$, $y \in (0, 2\pi)$ then $x = \dots$ & $y = \dots$
3. If α, β be the roots of the equation $ax^2 + bx + c = 0$ then the value of $\frac{a\alpha^2}{b\alpha + c} + \frac{a\beta^2}{b\beta + c}$ is equal to

MATCH THE COLUMN

Following question contains statements given in two columns, which have to be matched. The statements in **Column-I** are labelled as A, B, C and D while the statements in **Column-II** are labelled as p, q, r and s. Any given statement in **Column-I** can have correct matching with **ONE OR MORE** statement(s) in **Column-II**.

1. Consider the equation $x^2 + 2(a-1)x + a + 5 = 0$, where 'a' is a parameter. Match of the real values of 'a' so that the given equation has

Column-I		Column-II	
(A)	imaginary roots	(p)	$\left(-\infty, -\frac{8}{7}\right)$
(B)	one root smaller than 3 and other root greater than 3	(q)	$(-1, 4)$
(C)	exactly one root in the interval (1, 3) & 1 and 3 are not the root of the equation	(r)	$\left(-\frac{4}{3}, -\frac{8}{7}\right)$
(D)	one root smaller than 1 and other root greater than 3	(s)	$\left(-\infty, -\frac{4}{3}\right)$

ASSERTION & REASON

These questions contains, Statement-I (assertion) and Statement-II (reason).

- (A) Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.
 (B) Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for statement-I
 (C) Statement-I is true, Statement-II is false
 (D) Statement-I is false, Statement-II is true

1. **Statement-I** : If equation $ax^2 + bx + c = 0$; ($a, b, c \in \mathbb{R}$) and $2x^2 + 3x + 4 = 0$ have a common root, then $a : b : c = 2 : 3 : 4$.

Because

Statement-II : If $p + iq$ is one root of a quadratic equation with real coefficients then $p - iq$ will be the other root ; $p, q \in \mathbb{R}$, $i = \sqrt{-1}$

(A) A

(B) B

(C) C

(D) D

2. **Statement-I** : If $f(x)$ is a quadratic expression such that $f(1) + f(2) = 0$. If -1 is a root of $f(x) = 0$ then the other root is $\frac{8}{5}$.

Because

Statement-II : If $f(x) = ax^2 + bx + c$ then sum of roots $= -\frac{b}{a}$ and product of roots $= \frac{c}{a}$

- (A) A (B) B (C) C (D) D

3. **Statement-I** : If $a + b + c > 0$ and $a < 0 < b < c$, then the roots of the equation $a(x - b)(x - c) + b(x - c)(x - a) + c(x - a)(x - b) = 0$ are of both negative.

Because

Statement-II : If both roots are negative, then sum of roots < 0 and product of roots > 0

- (A) A (B) B (C) C (D) D

4. **Statement-I** : Let $(a_1, a_2, a_3, a_4, a_5)$ denote a re-arrangement of $(1, -4, 6, 7, -10)$. Then the equation $a_1x^4 + a_2x^3 + a_3x^2 + a_4x + a_5 = 0$ has at least two real roots.

Because

Statement-II : If $ax^2 + bx + c = 0$ and $a + b + c = 0$, (i.e. in a polynomial the sum of coefficients is zero) then $x = 1$ is root of $ax^2 + bx + c = 0$.

- (A) A (B) B (C) C (D) D

5. **Statement-I** : If roots of the equation $x^2 - bx + c = 0$ are two consecutive integers, then $b^2 - 4c = 1$.

Because

Statement-II : If a, b, c are odd integer then the roots of the equation $4abcx^2 + (b^2 - 4ac)x - b = 0$ are real and distinct.

- (A) A (B) B (C) C (D) D

COMPREHENSION BASED QUESTIONS

Comprehension # 1

If α, β, γ be the roots of the equation $ax^3 + bx^2 + cx + d = 0$. To obtain the equation whose roots are $f(\alpha), f(\beta), f(\gamma)$, where f is a function, we put $y = f(\alpha)$ and simplify it to obtain $\alpha = g(y)$ (some function of y).

Now, α is a root of the equation $ax^3 + bx^2 + cx + d = 0$, then we obtain the desired equation which is $a\{g(y)\}^3 + b\{g(y)\}^2 + c\{g(y)\} + d = 0$

For example, if α, β, γ are the roots of $ax^3 + bx^2 + cx + d = 0$. To find equation whose roots are

$$\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma} \text{ we put } y = \frac{1}{\alpha} \Rightarrow \alpha = \frac{1}{y}$$

As α is a root of $ax^3 + bx^2 + cx + d = 0$

$$\text{we get } \frac{a}{y^3} + \frac{b}{y^2} + \frac{c}{y} + d = 0 \Rightarrow dy^3 + cy^2 + by + a = 0$$

This is desired equation.

On the basis of above information, answer the following questions :

1. If α, β are the roots of the equation $ax^2 + bx + c = 0$, then the roots of the equation $a(2x + 1)^2 + b(2x + 1)(x - 1) + c(x - 1)^2 = 0$ are-

- (A) $\frac{2\alpha+1}{\alpha-1}, \frac{2\beta+1}{\beta-1}$ (B) $\frac{2\alpha-1}{\alpha+1}, \frac{2\beta-1}{\beta+1}$ (C) $\frac{\alpha+1}{\alpha-2}, \frac{\beta+1}{\beta-2}$ (D) $\frac{2\alpha+3}{\alpha-1}, \frac{2\beta+3}{\beta-1}$

2. If α, β are the roots of the equation $2x^2 + 4x - 5 = 0$, the equation whose roots are the reciprocals of $2\alpha - 3$ and $2\beta - 3$ is -

- (A) $x^2 + 10x - 11 = 0$ (B) $11x^2 + 10x + 1 = 0$
(C) $x^2 + 10x + 11 = 0$ (D) $11x^2 - 10x + 1 = 0$

EXERCISE - 04 [A]

CONCEPTUAL SUBJECTIVE EXERCISE

1. If α, β are the roots of the equation $x^2 - 2x + 3 = 0$ obtain the equation whose roots are $\alpha^3 - 3\alpha^2 + 5\alpha - 2, \beta^3 - \beta^2 + \beta + 5$.
2. If one root of the equation $ax^2 + bx + c = 0$ be the square of the other, prove that $b^3 + a^2c + ac^2 = 3abc$.
3. Show that if p, q, r & s are real numbers & $pr = 2(q + s)$, then at least one of the equations $x^2 + px + q = 0, x^2 + rx + s = 0$ has real roots.
4. Let a, b, c, d be distinct real numbers and a and b are the roots of quadratic equation $x^2 - 2cx - 5d = 0$. If c and d are the roots of the quadratic equation $x^2 - 2ax - 5b = 0$ then find the numerical values of $a + b + c + d$.
5. Find the product of the real roots of the equation, $x^2 + 18x + 30 = 2\sqrt{x^2 + 18x + 45}$
6. Find the range of values of a , such that $f(x) = \frac{ax^2 + 2(a+1)x + 9a + 4}{x^2 - 8x + 32}$ is always negative.
7. Find the values of 'a' for which $-3 < \frac{x^2 + ax - 2}{x^2 + x + 1} < 2$ is valid for all real x .
8. If the quadratic equations $x^2 + bx + ca = 0$ & $x^2 + cx + ab = 0$ have a common root, prove that the equation containing their other roots is $x^2 + ax + bc = 0$.
9. The equation $x^2 - ax + b = 0$ & $x^3 - px^2 + qx = 0$, where $b \neq 0, q \neq 0$, have one common root & the second equation has two equal roots. Prove that $2(q + b) = ap$.

Find the solutions of following inequations : (10 to 14)

10. $\frac{(x-2)(x-4)(x-7)}{(x+2)(x+4)(x+7)} > 1$
11. $(x^2 - x - 1)(x^2 - x - 7) < -5$.
12. $(x^2 - 2x)(2x - 2) - 9 \frac{2x - 2}{x^2 - 2x} \leq 0$
13. $\frac{1}{x-2} + \frac{1}{x-1} > \frac{1}{x}$
14. $\frac{20}{(x-3)(x-4)} + \frac{10}{x-4} + 1 > 0$

Find the solutions of following miscellaneous inequations : (15 to 20)

15. $|x^2 - 2x - 3| < |x^2 - x + 5|$
16. $x - 3 < \sqrt{x^2 + 4x - 5}$
17. $\log_{\frac{5}{8}} \left(2x^2 - x - \frac{3}{8} \right) \geq 1$
18. $\left(\frac{3}{4} \right)^{6x+10-x^2} < \frac{27}{64}$
19. $\log_{1/2} (x + 1) > \log_2 (2 - x)$.
20. $\log_x 2 \cdot \log_{2x} 2 \cdot \log_2 4x > 1$.
21. Find all values of a for which the inequality $(a + 4)x^2 - 2ax + 2a - 6 < 0$ is satisfied for all $x \in \mathbb{R}$.
22. Find all values of a for which both roots of the equation $x^2 - 6ax + 2 - 2a + 9a^2 = 0$ are greater than 3.

23. Find all the values of the parameter 'a' for which both roots of the quadratic equation $x^2 - ax + 2 = 0$ belong to the interval $(0, 3)$.
24. Find the values of K so that the quadratic equation $x^2 + 2(K - 1)x + K + 5 = 0$ has atleast one positive root.
25. If $a < b < c < d$ then prove that the roots of the equation $(x - a)(x - c) + 2(x - b)(x - d) = 0$ are real & distinct.
26. Two roots of a biquadratic $x^4 - 18x^3 + kx^2 + 200x - 1984 = 0$ have their product equal to (-32) . Find the value of k.

CONCEPTUAL SUBJECTIVE EXERCISE		ANSWER KEY	EXERCISE-4(A)
1. $x^2 - 3x + 2 = 0$	4. 30	5. 20	6. $a \in \left(-\infty, -\frac{1}{2}\right)$
7. $-2 < a < 1$	10. $(-\infty, -7) \cup (-4, -2)$	11. $(-2, -1) \cup (2, 3)$	12. $(-\infty, -1] \cup (0, 1] \cup (2, 3]$
13. $(-\sqrt{2}, 0) \cup (1, \sqrt{2}) \cup (2, +\infty)$	14. $(-\infty, -2) \cup (-1, 3) \cup (4, +\infty)$	15. $(-8, \infty)$	
16. $(-\infty, -5] \cup [1, \infty)$	17. $\left[-\frac{1}{2}, -\frac{1}{4}\right) \cup \left(\frac{3}{4}, 1\right]$	18. 7	
19. $-1 < x < \frac{1-\sqrt{5}}{2}$ or $\frac{1+\sqrt{5}}{2} < x < 2$	20. $2^{-\sqrt{2}} < x < 2^{-1}$; $1 < x < 2^{\sqrt{2}}$		
21. For all $a \in (-\infty, -6)$	22. For all $a \in (11/9, +\infty)$	23. $2\sqrt{2} \leq a < \frac{11}{3}$	
24. $K \leq -1$	26. $k = 86$		

EXERCISE - 04 [B]

BRAIN STORMING SUBJECTIVE EXERCISE

1. If one root of the quadratic equation $ax^2 + bx + c = 0$ is equal to the n^{th} power of the other, then show that $(ac^n)^{1/(n+1)} + (a^n c)^{1/(n+1)} + b = 0$.
2. Let $P(x) = x^2 + bx + c$, where b and c are integer. If $P(x)$ is a factor of both $x^4 + 6x^2 + 25$ and $3x^4 + 4x^2 + 28x + 5$, find the value of $P(1)$.
3. Find the true set of values of p for which the equation : $p \cdot 2^{\cos^2 x} + p \cdot 2^{-\cos^2 x} - 2 = 0$ has real roots.
4. If the coefficients of the quadratic equation $ax^2 + bx + c = 0$ are odd integers then prove that the roots of the equation cannot be rational number.
5. If the three equations $x^2 + ax + 12 = 0$, $x^2 + bx + 15 = 0$ and $x^2 + (a + b)x + 36 = 0$ have a common positive root, find a and b and the roots of the equations.
6. If the quadratic equation $ax^2 + bx + c = 0$ has real roots, of opposite sign in the interval $(-2, 2)$ then prove that

$$1 + \frac{c}{4a} - \left| \frac{b}{2a} \right| > 0.$$

7. Show that the function $z = 2x^2 + 2xy + y^2 - 2x + 2y + 2$ is not smaller than -3 .
8. For $a \leq 0$, determine all real roots of the equation $x^2 - 2a \mid x - a \mid - 3a^2 = 0$.
9. The equation $x^n + px^2 + qx + r = 0$, where $n \geq 5$ & $r \neq 0$ has roots $\alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n$.

Denoting $\sum_{i=1}^n \alpha_i^k$ by S_k .

- (a) Calculate S_2 & deduce that the roots cannot all be real.
 - (b) Prove that $S_n + pS_2 + qS_1 + nr = 0$ & hence find the value of S_n .
10. Find the values of 'b' for which the equation $2 \log_{\frac{1}{25}}(bx + 28) = -\log_5(12 - 4x - x^2)$ has only one solution.
 11. Solve the inequality : $\log_3 \frac{|x^2 - 4x| + 3}{x^2 + |x - 5|} \geq 0$

BRAIN STORMING SUBJECTIVE EXERCISE		ANSWER KEY	EXERCISE-4(B)
2. $P(1) = 4$	3. $[4/5, 1]$	5. $a = -7, b = -8 ; (3, 4) ; (3, 5)$ and $(3, 12)$	
8. $x = (1 - \sqrt{2})a$ or $(\sqrt{6} - 1)a$	9. (a) $S_2 = 0$, (b) $S_n = -nr$	10. $(-\infty, -14) \cup \{4\} \cup \left[\frac{14}{3}, \infty\right)$	
11. $x \leq \frac{-2}{3}, \frac{1}{2} \leq x \leq 2$			

EXERCISE - 05 [A]**JEE-[MAIN] : PREVIOUS YEAR QUESTIONS**

- If the roots of the equation $x^2 - 5x + 16 = 0$ are α, β and the roots of the equation $x^2 + px + q = 0$ are $(\alpha^2 + \beta^2)$ and $\frac{\alpha\beta}{2}$, then- [AIEEE-2002]
 - (1) $p = 1$ and $q = 56$
 - (2) $p = 1$ and $q = -56$
 - (3) $p = -1$ and $q = 56$
 - (4) $p = -1$ and $q = -56$
- If α and β be the roots of the equation $(x - a)(x - b) = c$ and $c \neq 0$, then roots of the equation $(x - \alpha)(x - \beta) + c = 0$ are - [AIEEE-2002]
 - (1) a and c
 - (2) b and c
 - (3) a and b
 - (4) $a + b$ and $b + c$
- If $\alpha^2 = 5\alpha - 3$, $\beta^2 = 5\beta - 3$ then the value of $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ (where $\alpha \neq \beta$) is- [AIEEE-2002]
 - (1) $19/3$
 - (2) $25/3$
 - (3) $-19/3$
 - (4) none of these
- The value of a for which one roots of the quadratic equation $(a^2 - 5a + 3)x^2 + (3a - 1)x + 2 = 0$ is twice as large as the other is [AIEEE-2003]
 - (1) $-2/3$
 - (2) $1/3$
 - (3) $-1/3$
 - (4) $2/3$
- If the sum of the roots of the quadratic equation $ax^2 + bx + c = 0$ is equal to the sum of the square of their reciprocals, then $\frac{a}{c}, \frac{b}{a}$ and $\frac{c}{b}$ are in [AIEEE-2003]
 - (1) geometric progression
 - (2) harmonic progression
 - (3) arithmetic-geometric progression
 - (4) arithmetic progression
- The number of real solutions of the equation $x^2 - 3|x| + 2 = 0$, is- [AIEEE-2003]
 - (1) 4
 - (2) 1
 - (3) 3
 - (4) 2
- The real number x when added to its inverse gives the minimum value of the sum at x equal to- [AIEEE-2003]
 - (1) 1
 - (2) -1
 - (3) -2
 - (4) 2
- Let two numbers have arithmetic mean 9 and geometric mean 4. Then these numbers are the roots of the quadratic equation- [AIEEE-2004]
 - (1) $x^2 + 18x - 16 = 0$
 - (2) $x^2 - 18x + 16 = 0$
 - (3) $x^2 + 18x + 16 = 0$
 - (4) $x^2 - 18x - 16 = 0$
- If $(1 - p)$ is a root of quadratic equation $x^2 + px + (1 - p) = 0$ then its roots are- [AIEEE-2004]
 - (1) 0, -1
 - (2) -1, 1
 - (3) 0, 1
 - (4) -1, 2
- If one root of the equation $x^2 + px + 12 = 0$ is 4, while the equation $x^2 + px + q = 0$ has equal roots, then the value of 'q' is- [AIEEE-2004]
 - (1) 3
 - (2) 12
 - (3) $49/4$
 - (4) 4
- If value of a for which the sum of the squares of the roots of the equation $x^2 - (a - 2)x - a - 1 = 0$ assume the least value is- [AIEEE-2005]
 - (1) 2
 - (2) 3
 - (3) 0
 - (4) 1
- If the roots of the equation $x^2 - bx + c = 0$ be two consecutive integers, then $b^2 - 4c$ equals- [AIEEE-2005]
 - (1) 1
 - (2) 2
 - (3) 3
 - (4) -2
- If both the roots of the quadratic equation $x^2 - 2kx + k^2 + k - 5 = 0$ are less than 5, then k lies in the interval- [AIEEE-2005]
 - (1) $[4, 5]$
 - (2) $(-\infty, 4)$
 - (3) $(6, \infty)$
 - (4) $(5, 6)$
- If the equation $a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x = 0$, $a_1 \neq 0$, $n \geq 2$, has a positive root $x = \alpha$, then the equation $na_n x^{n-1} + (n-1)a_{n-1} x^{n-2} + \dots + a_1 = 0$ has a positive root, which is- [AIEEE-2005]
 - (1) equal to α
 - (2) greater than or equal to α
 - (3) smaller than α
 - (4) greater than α

15. All the values of m for which both roots of the equation $x^2 - 2mx + m^2 - 1 = 0$ are greater than -2 but less than 4 , lie in the interval- [AIEEE-2006]
 (1) $-1 < m < 3$ (2) $1 < m < 4$ (3) $-2 < m < 0$ (4) $m > 3$
16. If the roots of the quadratic equation $x^2 + px + q = 0$ are $\tan 30^\circ$ and $\tan 15^\circ$, respectively then the value of $2 + q - p$ is- [AIEEE-2006]
 (1) 0 (2) 1 (3) 2 (4) 3
17. If x is real, then maximum value of $\frac{3x^2 + 9x + 17}{3x^2 + 9x + 7}$ is- [AIEEE-2006]
 (1) 1 (2) $\frac{17}{7}$ (3) $\frac{1}{4}$ (4) 41
18. If the difference between the roots of the equation $x^2 + ax + 1 = 0$ is less than $\sqrt{5}$, then the set of possible values of a is [AIEEE-2007]
 (1) $(-3, \infty)$ (2) $(3, \infty)$
 (3) $(-\infty, -3)$ (4) $(-3, -2) \cup (2, 3)$
19. The quadratic equations $x^2 - 6x + a = 0$ and $x^2 - cx + 6 = 0$ have one root in common. The other roots of the first and second equations are integers in the ratio $4:3$. Then the common root is [AIEEE-2008]
 (1) 1 (2) 4 (3) 3 (4) 2
20. If the roots of the equation $bx^2 + cx + a = 0$ be imaginary, then for all real values of x , the expression $3b^2x^2 + 6bcx + 2c^2$ is :- [AIEEE-2009]
 (1) Greater than $-4ab$ (2) Less than $-4ab$
 (3) Greater than $4ab$ (4) Less than $4ab$
21. If α and β are the roots of the equation $x^2 - x + 1 = 0$, then $\alpha^{2009} + \beta^{2009} =$ [AIEEE-2010]
 (1) -2 (2) -1 (3) 1 (4) 2
22. Let for $a \neq a_1 \neq 0$, $f(x) = ax^2 + bx + c$, $g(x) = a_1x^2 + b_1x + c_1$ and $p(x) = f(x) - g(x)$. If $p(x) = 0$ only for $x = -1$ and $p(-2) = 2$, then the value of $p(2)$ is: [AIEEE-2011]
 (1) 18 (2) 3 (3) 9 (4) 6
23. Sachin and Rahul attempted to solve a quadratic equation. Sachin made a mistake in writing down the constant term and ended up in roots $(4, 3)$. Rahul made a mistake in writing down coefficient of x to get roots $(3, 2)$. The correct roots of equation are: [AIEEE-2011]
 (1) $-4, -3$ (2) $6, 1$ (3) $4, 3$ (4) $-6, -1$
24. The equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ has : [AIEEE-2012]
 (1) exactly four real roots. (2) infinite number of real roots.
 (3) no real roots. (4) exactly one real root.

PREVIOUS YEARS QUESTIONS					ANSWER KEY			EXERCISE-5 [A]		
Que.	1	2	3	4	5	6	7	8	9	10
Ans.	4	3	1	4	2	1	1	2	1	3
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	4	1	2	3	1	4	4	4	4	1
Que.	21	22	23	24						
Ans.	3	1	2	3						

EXERCISE - 05 [B]**JEE-[ADVANCED] : PREVIOUS YEAR QUESTIONS**

1. Let a, b, c be real numbers with $a \neq 0$ and let α, β be the roots of the equation $ax^2 + bx + c = 0$. Express the roots of $a^3x^2 + abcx + c^3 = 0$ in terms of α, β .

[JEE 2001, Mains, 5 out of 100]

2. The set of all real numbers x for which $x^2 - |x + 2| + x > 0$, is

(A) $(-\infty, -2) \cup (2, \infty)$ (B) $(-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$

(C) $(-\infty, -1) \cup (1, \infty)$ (D) $(\sqrt{2}, \infty)$

[JEE 2002 (screening), 3]

3. If $x^2 + (a - b)x + (1 - a - b) = 0$ where $a, b \in \mathbb{R}$ then find the values of 'a' for which equation has unequal real roots for all values of 'b'.

[JEE 2003, Mains-4 out of 60]

4. (a) If one root of the equation $x^2 + px + q = 0$ is the square of the other, then

(A) $p^3 + q^2 - q(3p + 1) = 0$

(B) $p^3 + q^2 + q(1 + 3p) = 0$

(C) $p^3 + q^2 + q(3p - 1) = 0$

(D) $p^3 + q^2 + q(1 - 3p) = 0$

- (b) If $x^2 + 2ax + 10 - 3a > 0$ for all $x \in \mathbb{R}$, then

(A) $-5 < a < 2$

(B) $a < -5$

(C) $a > 5$

(D) $2 < a < 5$

[JEE 2004 (Screening)]

5. Find the range of values of t for which $2 \sin t = \frac{1 - 2x + 5x^2}{3x^2 - 2x - 1}$, $t \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

[JEE 2005(Mains), 2]

6. (a) Let a, b, c be the sides of a triangle. No two of them are equal and $\lambda \in \mathbb{R}$. If the roots of the equation $x^2 + 2(a + b + c)x + 3\lambda(ab + bc + ca) = 0$ are real, then

(A) $\lambda < \frac{4}{3}$

(B) $\lambda > \frac{5}{3}$

(C) $\lambda \in \left(\frac{1}{3}, \frac{5}{3}\right)$

(D) $\lambda \in \left(\frac{4}{3}, \frac{5}{3}\right)$

[JEE 2006, 3]

- (b) If roots of the equation $x^2 - 10cx - 11d = 0$ are a, b and those of $x^2 - 10ax - 11b = 0$ are c, d , then find the value of $a + b + c + d$. (a, b, c and d are distinct numbers)

[JEE 2006, 6]

7. (a) Let α, β be the roots of the equation $x^2 - px + r = 0$ and $\alpha/2, 2\beta$ be the roots of the equation $x^2 - qx + r = 0$. Then the value of 'r' is

(A) $\frac{2}{9}(p-q)(2q - p)$

(B) $\frac{2}{9}(q - p)(2p - q)$

(C) $\frac{2}{9}(q - 2p)(2q - p)$

(D) $\frac{2}{9}(2p-q)(2q - p)$

MATCH THE COLUMN :

(b) Let $f(x) = \frac{x^2 - 6x + 5}{x^2 - 5x + 6}$

Match the expressions / statements in **Column I** with expressions / statements in **Column II**.

Column I

(A) If $-1 < x < 1$, then $f(x)$ satisfies

(B) If $1 < x < 2$, the $f(x)$ satisfies

(C) If $3 < x < 5$, then $f(x)$ satisfies

(D) If $x > 5$, then $f(x)$ satisfies

Column II

(P) $0 < f(x) < 1$

(Q) $f(x) < 0$

(R) $f(x) > 0$

(S) $f(x) < 1$

[JEE 2007, 3+6]

ASSERTION & REASON :

8. Let a, b, c, p, q be real numbers. Suppose α, β are the roots of the equation $x^2 + 2px + q = 0$ and $\alpha, 1/\beta$ are the roots of the equation $ax^2 + 2bx + c = 0$, where $\beta^2 \notin \{-1, 0, 1\}$

STATEMENT-1 : $(p^2 - q)(b^2 - ac) \geq 0$

and

STATEMENT-2 : $b \neq pa$ or $c \neq qa$

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement-2 is **NOT** a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

[JEE 2008, 3 (-1)]

9. The smallest value of k , for which both the roots of the equation, $x^2 - 8kx + 16(k^2 - k + 1) = 0$ are real, distinct and have values at least 4, is

[JEE 2009, 4 (-1)]

10. Let p and q be real numbers such that $p \neq 0$, $p^3 \neq q$ and $p^3 \neq -q$. If α and β are nonzero complex numbers

satisfying $\alpha + \beta = -p$ and $\alpha^3 + \beta^3 = q$, then a quadratic equation having $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ as its roots is

[JEE 2010, 3]

(A) $(p^3 + q)x^2 - (p^3 + 2q)x + (p^3 + q) = 0$ (B) $(p^3 + q)x^2 - (p^3 - 2q)x + (p^3 + q) = 0$

(C) $(p^3 - q)x^2 - (5p^3 - 2q)x + (p^3 - q) = 0$ (D) $(p^3 - q)x^2 - (5p^3 + 2q)x + (p^3 - q) = 0$

11. Let α and β be the roots of $x^2 - 6x - 2 = 0$, with $\alpha > \beta$. If $a_n = \alpha^n - \beta^n$ for $n \geq 1$, then the value of $\frac{a_{10} - 2a_8}{2a_9}$ is

[JEE 2011]

(A) 1

(B) 2

(C) 3

(D) 4

12. A value of b for which the equations

$$x^2 + bx - 1 = 0$$

$$x^2 + x + b = 0,$$

have one root in common is -

[JEE 2011]

(A) $-\sqrt{2}$

(B) $-i\sqrt{3}$

(C) $i\sqrt{5}$

(D) $\sqrt{2}$

PREVIOUS YEARS QUESTIONS		ANSWER KEY	EXERCISE-5 [B]
1. $\gamma = \alpha^2\beta$ and $\delta = \alpha\beta^2$ or $\gamma = \alpha\beta^2$ and $\delta = \alpha^2\beta$		2. B	3. $a > 1$
4. (a) D ; (b) A	5. $\left[-\frac{\pi}{2}, -\frac{\pi}{10}\right] \cup \left[\frac{3\pi}{10}, \frac{\pi}{2}\right]$	6. (a) A; (b) 1210	
7. (a) D; (b) (A) P, R, S; (B) Q, S; (C) Q, S; (D) P, R, S		8. B	9. 2
10. B	11. C	12. B	